A model of planar polarity

Supplementary material to “Cell interactions and planar polarity in the abdominal epidermis of *Drosophila*”

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Our aim was to build a model for planar polarity that could explain the non-autonomy of *fz* mutant clones, since we believe it is the core of the problem. We have based our description on cellular automata. This is simple and perhaps close to the actual phenomena as cellular automata are discrete dynamical systems whose behaviour is specified in terms of local interactions and can be defined by a simple set of Rules. In the simplest situation, we can reduce an epithelium to a single array of cells. At the initial state each cell responds to a morphogen gradient and is assigned a given scalar value which we assume to be Fz activity (Rule 1). When the constant *s* is small a shallow gradient is produced. In successive states, the scalar value of each cell is determined by the iterative application of Rule 2. This Rule has two components; the first one is the response of each cell to the value at the previous state of its immediate neighbours; each cell compares (via Stan) its level of Fz activity with its neighbours and tries to adjust it to an average value. The second component is identical to that of Rule 1 since the cells are still perceiving the original morphogen. The degree to which each component influences the final scalar value is regulated by the constant *a*. Rule 2 can obviously apply only to cells in which the original scalar value is not null. If the latter is the case, the value remains 0 (Rule 3). A vectorial value is achieved by the comparison of the scalar values of each cell with its neighbours (Rules 4 and 5). A cell shows normal polarity only when the scalar value of the next cell is lower (or equal) and the value of the previous cell is equal (or

rules

One dimensional cellular automata with N cells. The cell at position n and time t has a scalar value $c_n^t \geq 0$

(1) At t = 0, cell n has the following scalar value:

$$c_n^0 = c_1^0 - n.s$$

where $s \geq 0$ and $c_1^0 >> s$

(2) If $c^0 > 0$ cell n has at t > 0 the following scalar value:

$$c_n^t = a \,.\, \frac{c_{n-1}^{t-1} + c_n^{t-1} + c_{n+1}^{t-1}}{3} \;+\; (1-a).c_n^0$$

(3) If $c^0 = 0$ a cell n has at t > 0 the following scalar value:

$$c_n^t = 0$$

(4) Cell n shows reversed polarity if any of the following statements is true:

$$c_{n-1}^t < c_n^t$$

$$c_n^t < c_{n+1}^t$$

(5) Cell n does not show a consisted polarity if:

$$c_{n-1}^t = c_n^t = c_n^t$$

(6) Otherwise cell n shows normal polarity

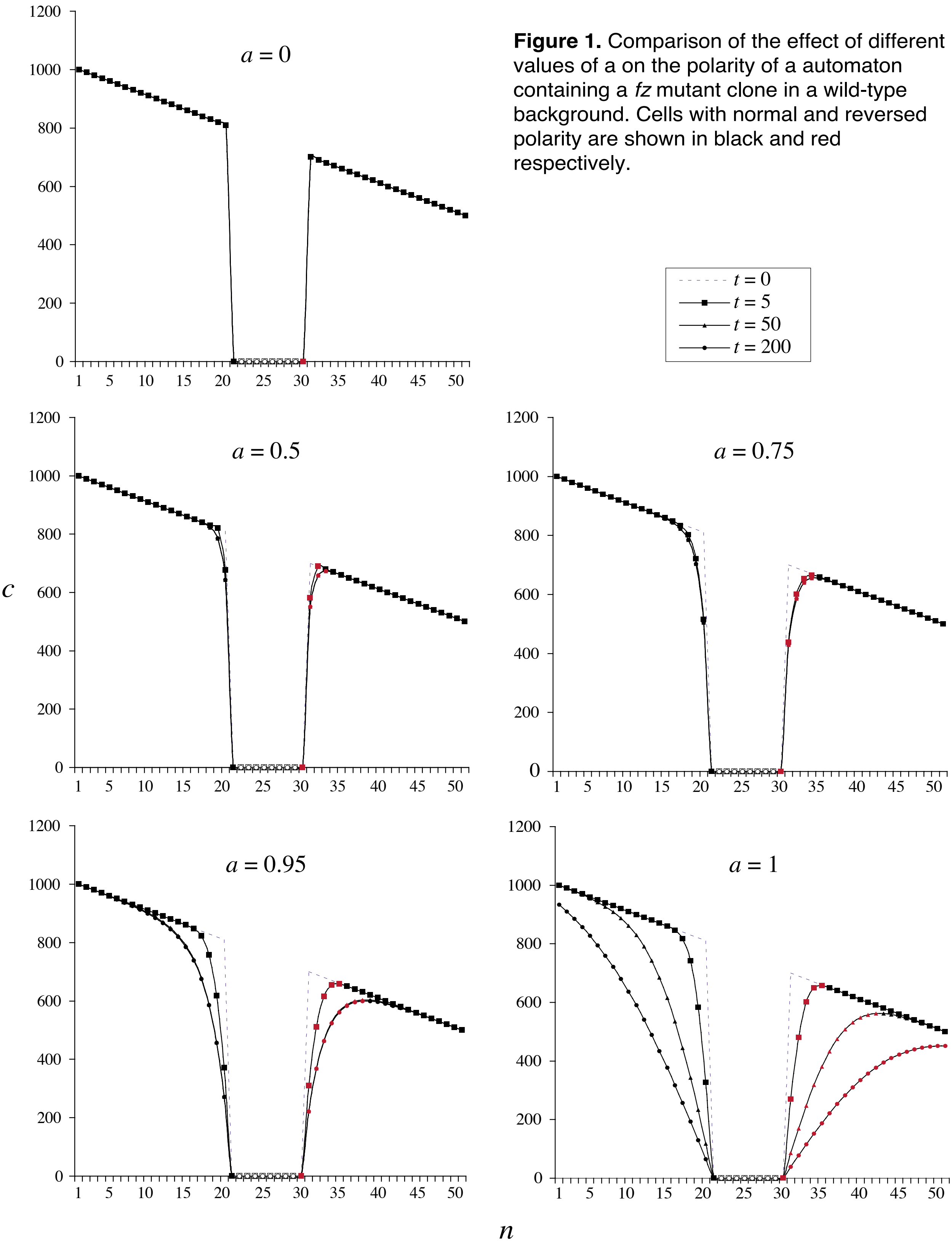

Figure 1. Comparison of the effect of different values of a on the polarity of a automaton containing a *fz* mutant clone in a wild-type background. Cells with normal and reversed polarity are shown in black and red respectively.

higher). If a cell has a lower scalar value than the next one, or has a higher scalar value than the previous cell, it shows reversed polarity.

Implications

We constructed an automaton of 50 cells in which the first cell had a value of 1000 at t=0 and we used s value of 10. This automaton behaves in a stable manner. It does not matter how many iterations of Rule 2 we apply or the value of a that we use, its cells have always the same value as in the original state (see Problems below), and therefore all the cells show the same normal polarity. The situation is different if some of the cells have a null scalar value (i.e. a *fz* mutant clone). We constructed an automaton similar to the previous one but with a clone of 10 cells (numbers 21 to 30), having a null initial scalar value. The behaviour of such an automaton varies enormously depending on the value of a (Figure1).

If the value of a is 0, the cells, after any number of iterations, have the same scalar value as in their initial state, but now one mutant cell (number 30) shows a reversed polarity since its value is lower than the adjacent wild-type cell (number 31), although the other neighbour is also mutant, i.e. it has also a null value (Rule 4).

If the value of a is 1, the scalar value of the wild-type cells is decreased. Starting with the cells closest to the mutant cells, the value of the cells tends to 0 when the number of iterations is increased (see Problems). This tendency is slowed down if the value of a is less than 1. We have compared the effect of using an a value of 0.95 versus 0.75. After five iterations there is virtually no difference. In both cases four wild-type cells, plus the mutant cell adjacent to them, show reversed polarity. However, after two hundred iterations the automaton with an a value of 0.95 has increased the number of wild-type cells with reversed polarity, whereas the automaton with an a value of 0.75 has still the same number, four, of reversed cells. In other words, the range of repolarization has reached an equilibrium between two opposing forces: on one hand, a cell tries to decrease its scalar value if it detects a neighbour with a lower value than itself, while on the other hand it is trying to keep a higher scalar value in responding to the information that it is receiving from the morphogen. The extent of reversed polarity is smaller if the value of a is reduced to 0.5. Three cells show reversed polarity after five iterations and only after fifty iterations of Rule 2 is the extent increased to four cells.

The position of reversed polarity, although not its extent, is changed if, instead of a mutant clone, we make a clone of cells that have, at the initial situation, a higher than normal scalar value (e.g. a clone over-expressing Fz. Figure 2). If we add 1,000 additional units to cells 21 to 30 an automaton as depicted in Figure 2 is obtained. After 200 iterations

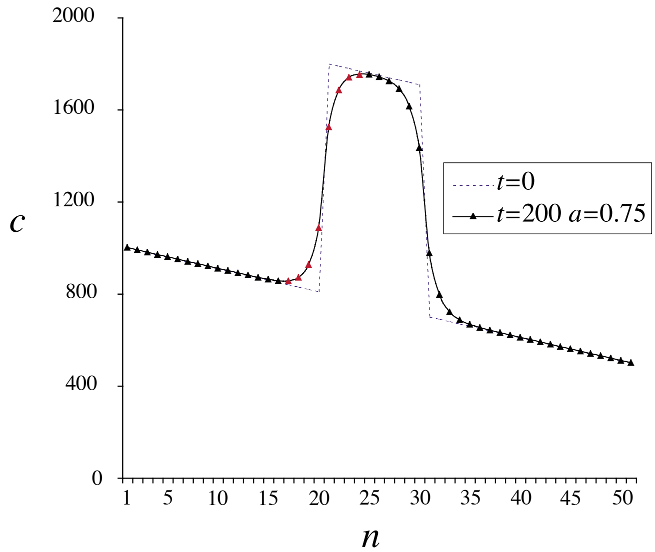

Figure 2. Effect of a Fz overexpressing clone on the polarity of a wild-type automaton. Cells with normal and reversed polarity are shown in black and red respectively.

and using an *a* value of 0.75, an equilibrium is reached with four wild-type cells in front of the clone having reversed polarity. However, and because the overexpressing cells have an initial value greater than zero, more than one cell within the clone also shows reversed polarity. We also examined the effect of mutant and over-expressing clones in a background in which the initial scalar value of the cells is maximized (i.e. *s* is 0). Clones of cells with a null initial scalar value in this automaton result in a similar behaviour to the one in Figure 1. One mutant cell at the edge of the clone also shows reversed polarity; however, the extent of repolarization outside the clone is increased (Figure 3), although this is mainly due to a long tail of small scalar differences. A similar behaviour is produced by an overexpressing clone in this automaton. There is a similar increase in the extent of the repolarization, although its position is at the other side of the clone (Figure 4).

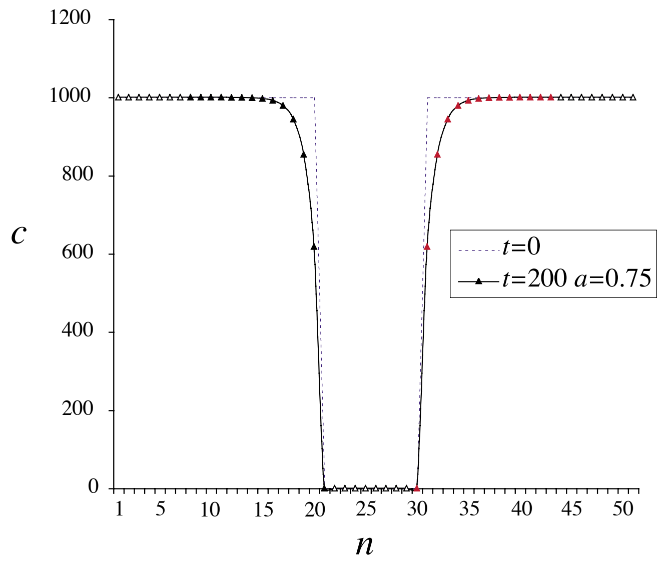

Figure 3. Effect of a *fz* clone on the polarity of a flat automaton. Cells with normal and reversed polarity are shown in black and red respectively.

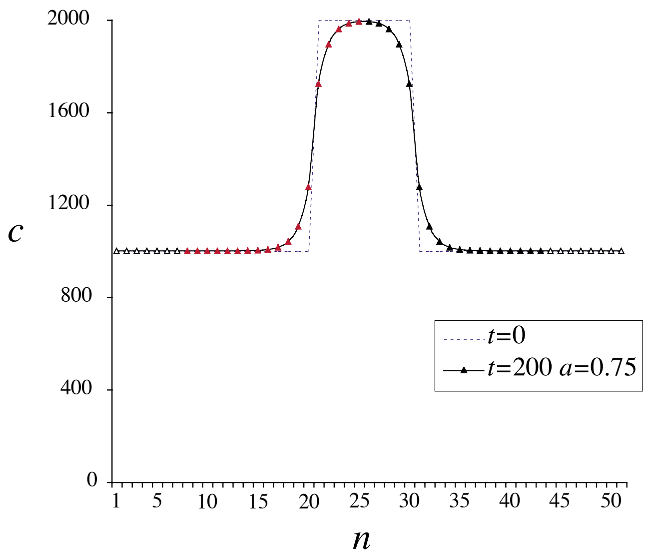

Figure 4. Effect of a Fz overexpressing clone on the polarity of a flat automaton. Cells with normal and reversed polarity are shown in black and red respectively.

Problems

The automata that we have described have a constant number of cells. Obviously, this is not the case in a proliferating epithilium. Therefore, the Rules have to accommodate the intercalation of new cells. As new cells are intercalated, they should acquire a scalar value that depends on both the position in which they are intercalated into the epithelium and the values of their neighbours. The first and last cell of the automata bring about another problem, since each of them has only one neighbour. Although theoretically we can waive this problem by considering automata of an infinite number of cells, that is, again, not the case in an epithelium.